

### 3RD “MATH-CHALLENGE” PRIZE PROBLEMS

ABSTRACT. Towards the celebration of the upcoming International Day of Mathematics on Monday 1th March, 2021, we pose the following problem set of mathematical challenges with prizes ranging from GHC 100.00 to GHC 300.00 to be awarded for each **rigorous** solution to the problems. The deadline for submitting a correct solution is 20:00 GMT Sunday 14th March, 2021. Detailed rules governing the problems and the prizes can be found at the announcement page at <https://math.knust.edu.gh>.

#### 1. ELEMENTARY PROBLEM

This is worth GHC 100.00 for a correct solution.

**MaTH-Challenge Elementary Problem** (Limits of sequences). *Let  $f(m, n)$  be a real-valued double-sequence, i.e. a function from the Cartesian product  $\mathbb{N} \times \mathbb{N}$  of the natural numbers to the real numbers. Show that*

$$\limsup_{m \rightarrow \infty} \liminf_{n \rightarrow \infty} f(m, n) \leq \liminf_{n \rightarrow \infty} \limsup_{m \rightarrow \infty} f(m, n),$$

whenever the limit exists.

#### 2. INTERMEDIATE PROBLEM

This is worth GHC 200.00 for a correct solution.

**MaTH-Challenge Intermediate Problem** (Pythagorean-like Triples). *The famous Pythagorean Triples are integers that are solutions to the equation*

$$x^2 + y^2 = z^2$$

whose solution can be parametrized as

$$x = m^2 - n^2 \quad y = 2mn \quad z = m^2 + n^2,$$

where  $m$  and  $n$  are integers. Deduce, rigorously, the parametric integer solutions to the equation

$$x^2 + dy^2 = z^2$$

where  $d$  is constant integer that is not a square.

### 3. ADVANCED PROBLEM

This is worth GHC 300.00 for a correct solution.

**MaTh-Challenge Advanced Problem** (Mathematics Application).  
*A solid spherical ball in 3-dimensions is to be constructed and an arbitrary number of points, excluding the centre, are to be selected inside the ball. What should be the least upper bound of all possible angles between the chosen points, measured from the centre, such that there will always be a way to divide the ball exactly into two hemispheres through the centre, with the points lying ONLY in one of the hemisphere.*

### 4. RULES FOR THE MATH-CHALLENGE PRIZE

The MaTh-Challenge Problems are *open* to all undergraduate students, and solutions should be sent to both email addresses below. However, detailed rules governing the problems and the prizes can be found at the announcement page at: <https://math.knust.edu.gh>.

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