"MATH-CHALLENGE" PRIZE PROBLEMS: WEEK 3 - APPLICATIONS OF MATHEMATICS

Abstract. This is the third problem set of a series of mathematical challenges with a prize of GHC 500.00 to be awarded per **rigorous** solution to each of the problems.

1. Problem Statement 3A

In the following, the notation $\frac{\partial \xi}{\partial x}$ denotes partial derivative of ξ with respect to x. The first of this week's MaTH-Challenge problems is:

MaTH-Challenge Problem 3A. *In modeling the elongation $\xi(x,t)$ of a travelling wave at point x and time t, the following non-linear partial differential equation was obtained:*

$$a\frac{\partial \xi}{\partial t} + b\xi\frac{\partial \xi}{\partial x} + c\frac{\partial^3 \xi}{\partial x^3} = 0$$

for some constants a, b, c. Assuming, say, a trial solution of the form $\xi(x,t) = f(x-vt) = f(\zeta)$ with the requirement that as $|x| \to \infty$ then $f \to 0$, $\frac{df}{d\zeta} \to 0$, and $\frac{d^2 f}{d\zeta^2} \to 0$, demonstrate an exact solution to the partial differential equation above.

2. Problem Statement 3B

The second problem of this week's MaTH-Challenge problem is:

MaTH-Challenge Problem 3B. *In an economic modeling of profit, revenue and cost, the production function $p(x_1, x_2, \ldots, x_n)$ of producing x_k units of item I_k (for all $1 \leq k \leq n$) was obtained as follows:*

$$p(x_1, x_2, \ldots, x_n) = \sum_{k=1}^{n} \alpha_k x_k^2 + \sum_{j=1}^{n-1} \sum_{k=j+1}^{n} \beta_{j,k} x_j x_k$$

for some constants $\{\alpha_k, \beta_{j,k}\}_{j,k\geq 1}$ to be determined. Demonstrate requirements on the constants that would ensure that the production level is (i) a minimum, and (ii) a maximum.

3. Rules for the MaTH-Challenge Prize

Solutions should be sent to the email address below. **However**, detailed rules can be found at the announcement page at the website **https://math.knust.edu.gh**. Two solutions were received for the Pure Mathematics problems, and awardees have been announced. (No solutions were received for Statistics and Probability problems).

E-mail address: jezearn@knust.edu.gh (Room SF 22)