"MATH-CHALLENGE" PRIZE PROBLEMS: WEEK 2 - STATISTICS AND PROBABILITY

Abstract. This is the second problem set of a series of mathematical challenges with a prize of GHC 500.00 to be awarded per **rigorous** solution to each of the problems. (These problems will be posted at announcement section of **https://math.knust.edu.gh**).

1. Problem Statement 2A

Denote by $\mathbb{R}^n$ an n-dimensional space of random variables, and for any $X = (x_1, \ldots, x_n) \in \mathbb{R}^n \backslash \{\mathbf{0}\}$ and $Y = (y_1, \ldots, y_n) \in \mathbb{R}^n \backslash \{\mathbf{0}\}$, define their Pearson's Product Moment Correlation Coefficient $\texttt{Corr}(X, Y)$ as

$$\texttt{Corr}(X, Y) := \frac{x_1 y_1 + \cdots + x_n y_n}{\sqrt{x_1^2 + \cdots + x_n^2}\sqrt{y_1^2 + \cdots + y_n^2}}\,.$$

The first of this week's MaTH-Challenge problems is as follows:

MaTH-Challenge Problem 2A. *Let $\mathcal{S} \subset \mathbb{R}^n \setminus \{\mathbf{0}\}$ be a subset of random variables such that*

$$\texttt{Corr}(X, Y) > -\frac{1}{n}\,, \quad \forall X, Y \in \mathcal{S}\,.$$

Show that there exists a random variable $Z \in \mathbb{R}^n \setminus \{\mathbf{0}\}$ such that

$$\texttt{Corr}(X, Z) > 0\,, \quad \forall X \in \mathcal{S}\,.$$

2. Problem Statement 2B

In the following, we denote by $\chi_k^2(\lambda)$ the non-central chi-square distribution, with $k > 0$ being the degree of freedom and λ its non-centrality parameter. The second problem of this week's MaTH-Challenge is:

MaTH-Challenge Problem 2B. *Let X be a random variable and let $\chi_k^2(\lambda)$ be its non-central chi-square distribution. Demonstrate (i) the characteristic function of Y, (ii) the expectation and variance of Y, and (iii) the expectation and variance of non-central F-distribution.*

3. Rules for the MaTH-Challenge Prize

Solutions should be sent to both email addresses below. **However**, you are entreated to kindly obtain further detailed information on the rules at the announcement section at **https://math.knust.edu.gh**. Two solutions were received for last week's Pure Mathematics problems, and awardees will be announced on the website by close of day.

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